Dynamics of *Pinus* wood prices for different timber assortments: comparison of stochastic processes

Photo 1.
Planted forests *Pinus* in Brazil.
Photo R. A. Munis, J. C. Martins, D. A. Camargo, D. Simões.

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RÉSUMÉ

Dynamique du prix des bois de *Pinus* selon différents assortiments d'essences : comparaison entre processus stochastiques

La compréhension de la dynamique des prix du marché pour le bois de *Pinus* est une condition préalable aux décisions stratégiques concernant les plans d'investissement forestier puisque, du point de vue du marché, le risque exogène d'un projet dépend des assortiments d'essences forestières. Il faut donc connaître le processus stochastique qui représente la meilleure façon d'évaluer l'actif sous-jacent. A l'aide de tests économétriques, la présente étude vise à comparer le mouvement brownien fractionnaire et le mouvement brownien géométrique pour déterminer le modèle stochastique qui représente le mieux le comportement du prix du bois de *Pinus* provenant de forêts plantées dans l'État de Santa Catarina, au Brésil, afin d'évaluer l'actif sous-jacent et les options réelles intrinsèques aux projets d'investissement forestier. Les séries chronologiques de prix, pour la période allant de juin 2017 à juillet 2019, concernent trois assortiments de bois de *Pinus* utilisés pour de multiples produits. Les tests économétriques recommandés pour analyser les séries chronologiques portaient sur la normalité des données, la tendance, l'autocorrélation, la stationnarité et l'estimation différentielle fractionnelle. Les séries chronologiques ont ensuite été modélisées au moyen de processus stochastiques conformément aux tests économétriques. Les séries chronologiques ont indiqué un comportement normal, la présence d'une tendance positive et la non-stationnarité des données. En outre, une mémoire longue a été trouvée dans toutes les séries. Le mouvement brownien fractionnaire s'est avéré être le processus stochastique le plus approprié pour modéliser les prix de trois assortiments de bois forestiers, étant donné les caractéristiques non stationnaires et la mémoire longue des séries chronologiques pour les prix du bois de *Pinus*.

Mots-clés : évaluation des actifs, processus stochastique, mouvement brownien fractionnaire, mouvement brownien géométrique, test économétrique.

ABSTRACT

Dynamics of *Pinus* wood prices for different timber assortments: comparison of stochastic processes

Understanding the dynamics of market prices for *Pinus* wood is a prerequisite for strategic decisions concerning forest investment plans since, in terms of the market, the exogenous risk to a project depends on timber assortments. The stochastic process that represents the best way of pricing the underlying asset therefore needs to be known. This study set out to compare Fractional Brownian Motion and Geometric Brownian Motion, through econometric tests, to understand the stochastic model that best represents the price behaviour of *Pinus* wood from planted forests in the state of Santa Catarina, Brazil, for the pricing of the underlying asset and valuation of the real options intrinsic to forest investment projects. The time series of prices, for the period from June 2017 to July 2019, relate to three assortments of *Pinus* wood used for multiple products. The recommended econometric tests to analyse the time series were for normality of the data, trend, autocorrelation, stationarity and fractional differential estimation. The time series were then modelled by means of stochastic processes in line with the econometric tests. The time series showed normal behaviour and indicated the presence of a positive trend and non-stationarity in the data. In addition, a true long memory was found in all series. Fractional Brownian Motion proved to be the most suitable stochastic process for modelling the prices of three forest timber assortments, given the non-stationary characteristics and true long memory of the time series for *Pinus* wood prices.

Keywords: asset pricing, stochastic process, fractional Brownian motion, geometric Brownian motion, econometric test.

RESUMEN

Dinámica de los precios de la madera de *Pinus* para distintos surtidos de madera: comparación de procesos estocásticos

Comprender la dinámica de los precios de mercado de la madera *de Pinus* es un requisito previo para las decisiones estratégicas relativas a los planes de inversión forestal, ya que, en términos de mercado, el riesgo exógeno de un proyecto depende de los surtidos de madera. Por lo tanto, es necesario conocer el proceso estocástico que representa la mejor forma de fijar un precio al activo subyacente. Este estudio se propuso comparar el movimiento browniano fraccional y el movimiento browniano geométrico, a través de pruebas econométricas, para entender el modelo estocástico que mejor representa el comportamiento del precio de la madera de *Pinus* de bosques plantados en el estado de Santa Catalina, Brasil, con el fin de evaluar los precios del activo subyacente y las opciones reales intrínsecas a los proyectos de inversión forestal. Las series temporales de precios, para el periodo comprendido entre junio de 2017 y julio de 2019, se refieren a tres surtidos de madera de *Pinus* utilizados para múltiples productos. Las pruebas econométricas recomendadas para analizar las series temporales fueron la normalidad de los datos, la tendencia, la autocorrelación, la estacionalidad y la estimación diferencial fraccional. A continuación, las series temporales se modelaron mediante procesos estocásticos de acuerdo con las pruebas econométricas. Las series temporales mostraron un comportamiento normal e indicaron la presencia de una tendencia positiva y la no estacionariedad de los datos. Además, se encontró una memoria a largo plazo en todas las series. El movimiento browniano fraccional resultó ser el proceso estocástico más adecuado para modelar los precios de tres surtidos de madera forestal, dadas las características no estacionarias y la memoria a largo plazo de las series temporales de los precios de la madera de *Pinus*.

Palabras clave: fijación de precios de activos, proceso estocástico, movimiento browniano fraccional, movimiento browniano geométrico, test econométrico.

Introduction

Stochastic modelling of wood prices provides information to improve the commercial use and strategic planning of companies in the forestry sector and, consequently, enables decision making based on reliable models that do not undervalue or overestimate the expectations of financial revenues.

Among the alternatives of stochastic models for the real options valuation, there is the use of Geometric Brownian Motion (GBM) for price modelling (Berk and Podhraski, 2018) with a linear stochastic solution using the Itô's lemma (Maeda and Watts, 2019). In addition, the GBM simulates the prices of the underlying asset following a random walk with constant volatility (Ardian and Kumral, 2020), assuming a lognormal probability distribution when the ratio between the expected price and the present price has independence from the past price history (Angstmann *et al*., 2019).

Another stochastic model with useful characteristics for price modelling is the Fractional Brownian Motion (FBM), which according to Gajda and Wyłomańska (2014), is a generalization of classic Brownian Motion and, in the understanding of Xu *et al*. (2019), this model belongs to the family of Gaussian processes and assumes that increments are not necessarily independent. FBM is also a long memory process (Kozachenko *et al*., 2015) and, according to Le Breton (1998), it can be considered one of the simplest stochastic processes that exhibit long-range dependence.

It is noteworthy that both stochastic models are Gaussian, however, the GBM has independent increments, while the FBM has correlated increments in series, thus, the historical trajectory of the process becomes fundamental when predicting its future evolution (Wang *et al*., 2018).

Insley (2002) demonstrated that the assumption that wood prices follow a process of reversion to the average, instead of GBM, can make a significant difference in the valuation of forest stand and the ideal harvest time. Likewise, FBM may be more appropriate than GBM to model timber prices over time because the fractional integration is the discrete counterpart of FBM (Manley and Niquidet, 2017).

In the analysis of investment projects, the stochastic process to be used in price modelling is fundamental in obtaining the value of flexibility. Furthermore, according to Matias *et al*. (2005), price modelling is one of the most important and delicate challenges of the process, as it influences future revenue that is commonly based on the behaviour of prices over time, which combined with econometric analysis, provide statistical information about the behaviour of the market under analysis (Olsson *et al*., 2011).

In this context, the time series of *Pinus* wood prices presents an additional component of complexity since they are normally sold in assortments. In agreement with Silva *et al*. (2017) and Kohler *et al*. (2015), the characterization of assortments describes the classification of logs according to their quality, dimensions, possibilities of use, and economic value.

Thus, it is justified to analyse more than one time series of *Pinus* wood prices since each forest assortment has different prices. Hence, the stochastic models used to forecast prices provide more plausible information, for example, the timing of the timber harvest and the application of capital for the expansion of planted forests.

However, Cordeiro *et al*. (2010) and Niquidet and Sun (2012) point out that despite the expectation of the price of wood being a fundamental factor for adequate forest management and for informing the development of the wood market, there is a lack of adjustment of models in relation to the stochastic properties of prices.

Therefore, in line with Manley and Niquidet (2010), there is a lack of consensus on how to specify the stochastic process of wood prices. According to Plantinga (1998) forest owners could more accurately value planted forests from the approach of stochastic price variations. That said, the need to select stochastic models to characterize the series of wood price and to pricing the underlying asset for *Pinus* assortments planted in Brazil is confirmed.

In this perspective, we analyse the prices of *Pinus* wood for different forest assortments in order to identify the behaviour of the wood prices time series and, therefore, compare Fractional Brownian Motion and Geometric Brownian Motion, and then, indicate the stochastic model recommended for the underlying asset pricing and the intrinsic real options valuation of forest investment projects.

Material and Methods

Study support

We carried out the econometric characterization of *Pinus* wood price time series from planted forests in the state of Santa Catarina, Brazil, according to data provided by the Centre for Socioeconomics and Agricultural Planning (2019). We used a time series of the monthly prices of *Pinus* wood from June 2017 to July 2019. The prices were derived from the average obtained from producers for different forest assortments, or rather, which supply wood to the processing industries with a view to obtaining multiproducts, characterized as:

Cellulose: first forest assortment of a *Pinus* plantation with diameter at breast height of the standing tree ranging between 8 cm and 17 cm, used as a raw material for the manufacture of cellulose pulp;

Sawmill I: second forest assortment of a *Pinus* plantation with diameter range of the standing tree defined between 18 cm to 24 cm, used as sawn wood;

Sawmill II: third forest assortment of a *Pinus* plantation with tree diameter range between 25 cm to 34 cm, for the supply of sawn wood.

Econometric tests of the price time series

To improve the accuracy of the projected wood prices, note the importance of using temporary wood price limits, which, by estimate, provide the application of stochastic models used with a price trajectory.

In this sense, in order to verify the behaviour of the price of wood, we performed econometric tests, using software R, to confirm normality, trend, autocorrelation, stationarity and the fractional differential estimate or true long memory of the wood price time series from the *Pinus* assortments.

Normality of the price time series data

The normality of the data should be the first parameter analysed, especially for economic data, since it acts as an indication in the selection of parametric or non-parametric tests, and, thus, it reduces large deviations and increases the accuracy of the results (Chen and Cheng, 2007).

Therefore, we test the normality of the data and the residuals of the regression analysis using the Jarque Bera test (Jarque and Bera, 1980) with a null hypothesis of normal data distribution at the 5% significance level.

Trend of the price time series

Trend time series models have gained attention in the past two decades due to many applications in corporate finance (Cai, 2007). When we approach the application of time series of prices, it is essential to verify the presence of trends and, therefore, their configuration, whether increasing, decreasing or stationary (Antunes and Cardoso, 2015). For this, we applied the Cox Stuart test (Cox and Stuart, 1955) with a null hypothesis that the time series does not trend to the 5% significance level.

Autocorrelation of the price time series

Through the linear regression of linearized prices at the present moment (t) with the past price (t–1), we verified whether the behavior of prices has any memory, or whether it actually meets Markov's property (Matias *et al*., 2005). Therefore, it is highly recommended to confirm the regression adjustment by analyzing the normality of the residuals.

Trend of the price time series

Once the normality of the data was indicated, we opted for applying the Dickey-Fuller parametric test (Dickey and Fuller, 1979), with a null hypothesis of unit root, I (1), at the level of 5% significance. In addition, we also opted for the application of the KPSS test (Kwiatkowski *et al*., 1992), with null hypothesis of stationarity, I (0) at the level of 10% of significance since it allows to verify the presence of true long memory in the time series.

Fractional differential estimate

Usually, economic time series exhibit cycles with orders of several quantities, thus, the spectra of a sample of these series do not show an accentuated pure period, but a spectral density (Mandelbrot and Ness, 1968).

Thus, using the Geweke and Porter-Hudak (GPH) estimator that returns Hurst's H parameter of a time series suggested by Geweke and Porter-Hudak (1983) we obtained the fractional differential parameter (d) of the linearized time series.

Modelling the price of *Pinus* wood

Supported by the econometric tests of the three *Pinus* assortments' wood price time series, we performed the modelling of the underlying asset, in order to estimate the dynamics of this price through the FBM and the GBM.

In this way, we estimated the values of drift (α) and volatility (σ) in annual terms, using the average and standard deviation of the series of linearized returns. Thus, for the valuation of real options, we model the price of *Pinus* wood for the three assortments by applying the stochastic model FBM (Equation 1) as described by Gu *et al*. (2012).

$$P_t = P_{t-1} e^{\left(\alpha t - \frac{\sigma^2 t^{2H}}{2}\right)} e^{\alpha\sqrt{\Delta t} W^H} \quad \text{[Equation 1]}$$

where: P_{t-1} is the initial condition; P_t is the price of wood in the subsequent period; α is the constant growth rate (drift); σ is the standard deviation (volatility); *t* is the period; *H* is the Hurst coefficient; W^H is the fractional Gaussian noise; Δ*t* is the time increment.

Furthermore, based on the inputs of the FBM stochastic model, that is, on drift and volatility, we model the price of *Pinus* wood, when subject to adjustment, using the GBM stochastic model (Equation 2) according to Ossenbruggen and Laflamme (2019).

$$P_t = P_{t-1} e^{\left(\alpha - \frac{\sigma^2}{2}\right)\Delta t} e^{\sigma \varepsilon \sqrt{\Delta t}} \quad \text{[Equation 2]}$$

where: ε is the random error with standard normal distribution.

In addition, the determination of the best fit for wood price modelling was done through the Mean Absolute Error and the root mean square error (RMSE), both used as performance indicators. Associated, the coefficient of determination (R^2) was used as a performance measure.

Results

For the price series used, we accept the assumptions of normality and trend of each assortment of *Pinus* wood prices (table I), with a view to valuing their Real Options. It means the incorporation of managerial flexibilities in forest investment projects. Moreover, we obtained the trend of price time series to check the discrepancy in relation to observed prices.

Table I.
Normality and trend of the *Pinus* wood prices time series for different forest assortments.

Forest assortment	Data normality	Trend in the *Pinus* wood prices time series
Cellulose	0.705	0.0009*
Sawmill I	0.324	0.0009*
Sawmill II	0.169	0.0117*

* At the 5% level of significance.

Table III.
Stationarity tests, GPH estimate, fractional memory and Hurst coefficient of *Pinus* wood prices for different forest assortments.

Forest assortment	Dick-Fuller (Pt)	KPSS (Pt)	d (Pt)	H (ln(Pt)-ln(Pt-1))
Cellulose	-2.4143*	0.1**	0.55	0.05
Sawmill I	-2.4906*	0.1**	1.01	0.51
Sawmill II	-2.4699*	0.1**	1.32	0.82

*At the 5% level of significance.
**p-value = 0.1 represents p-value >= 10%.

After the application of the natural logarithm (*ln*) in the *Pinus* wood price time series, since as the errors of the stochastic models are normally distributed, the variable could assume negative values, which does not match the economic approach, we obtained the autocorrelation found through regression analysis (table II). Consequently, the adjustment confirmed by the Jarque Bera test, with the normality of the residuals, however, occurred only for the cellulose assortment.

In view of this, through the Dickey-Fuller and KPSS tests, we found that the time series of *Pinus* is non-stationary, except for the current time dependence, confirmed through the GPH estimate (table III).

In view of the values we obtained for Hurst's *H* parameters from the time series of the *Pinus* wood price and, also, the presence of true long memory in all-time series, since through this KPSS test it is possible to verify if the second derivative of a series follows a process I (0), identifying a true long memory against spurious long memory (Niquidet and Sun, 2012), we performed three simulations of wood price modelling through the FBM for the Cellulose forest assortment (figure 1a), Sawmill I (figure 1b) and Saw mill II (figure 1c).

In view of this, the GBM has a Hurst index of 0.5 and is considered a particular type of FBM, therefore, Manley and Niquidet (2017) indicate that there are no material differences in modelling the asset through GBM compared to FBM, when the Hurst index is 0.49. Given the same amplitude of the Hurst index found by these authors, that is, 0.51 for the Sawmill II forest assortment, we also performed three simulations, but modelled using the GBM (figure 1d).

Thus, it was inferred the best performance of *Pinus* wood price modelling in the different assortments evaluated by the FBM, mainly due to the fact that the MAE and the RMSE are inferior to the GBM, with the exception of Sawmill II. However, the coefficient of determination allowed inferring that when modelled by the FBM, it can better explain the variation of the data than obtained by the GBM (table IV).

Table IV.
Model performance evaluation.

Multiproduct	Model	R^2	MAE (USD)	RMSE (USD)
Cellulose	FBM	0.53	0.85	0.91
	GBM	0.33	0.94	1.01
Sawmill I	FBM	0.64	1.07	1.31
	GBM	0.36	1.27	1.58
Sawmill II	FBM	0.73	2.37	2.68
	GBM	0.19	1.42	2.09

Table II.
Regression analysis results of the of *Pinus* wood prices for different forest assortments.

Forest assortment	R^2 adjusted	*t-value*	Characteristic line equation	Data normality
Cellulose	0.83	10.43	$\ln(P_t) = 0.081 + 0.8663.\ln(P_{t-1})$	0.62
Sawmill I	0.83	11.00	$\ln(P_t) = 0.219 + 0.9037.\ln(P_{t-1})$	$1.72.e^{-06}$*
Sawmill II	0.75	8.16	$\ln(P_t) = 0.330 + 0.8839.\ln(P_{t-1})$	$1.28.e^{-07}$*

* At the 5% level of significance.

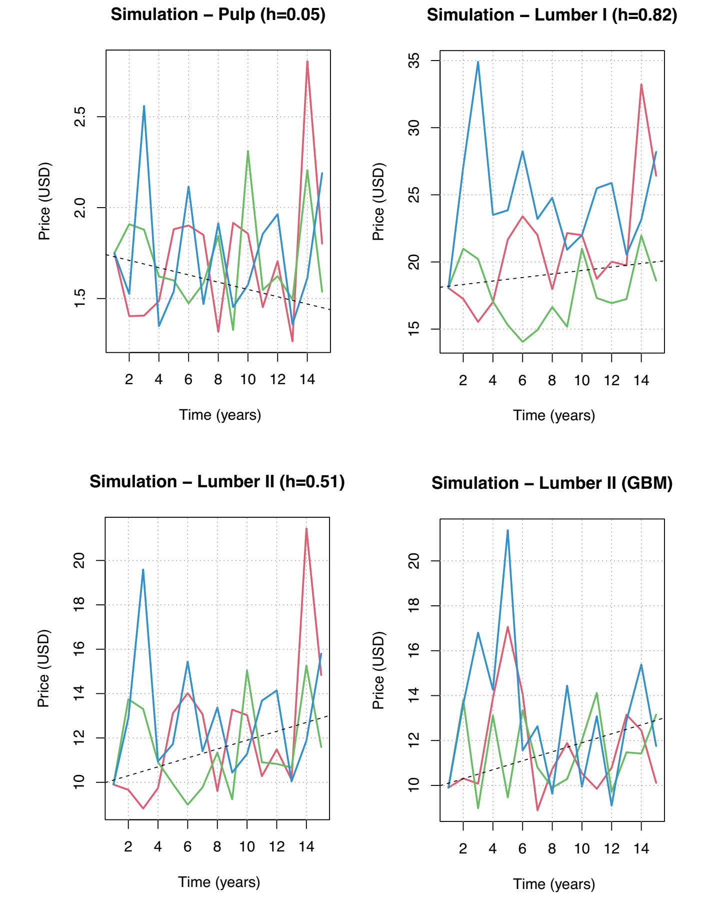

Figure 1.
Representation of the simulations with the *Pinus* wood price modelling for different forest assortments. The solid lines represent the results of the three different simulations performed for each condition. The dotted line is the series drift.

Discussion

Pinus planted forests emerge due to the increased demand for wood, used as raw material for various segments, providing economic and social benefits. In this sense, when we compare *Pinus* wood price series modelling criteria, we are providing inputs to managers so that they can discern among the most viable options for providing effective property rights, insurance, stocking price policy, governance and forest management.

Our econometric treatment of the *Pinus* wood price time series evidenced a normal distribution of price data over two years of observation. Furthermore, trend parameters were identified, which underlie the organizational strategy of forest-based transformation industries, in addition to enhancing the economic results of the projected time horizons.

Zhang *et al*. (2015) point out that planted forests develop in response to economic instruments, which require an understanding of this scenario, as well as its evolution. Therefore, we verified, through the Dickey-Fuller and KPSS test, the temporal dependence in the evolution of *Pinus* wood prices for the assortments used in the production of pulp and wood splitting in Sawmill I Sawmill II.

Niquidet and Manley (2007) add that fluctuations in these prices are intrinsically related to market volatility and trends. Therefore, the proper management of the *Pinus* genus, added to the application of taper models provided, in the same planted forest, different recipes, confirming the study developed by Kohler *et al*. (2015), because the same forest individual can generate the sale of several by-products.

Therefore, the generation of future income must be adequate for silvicultural management. In this way, we characterize the use of forest assortments of the *Pinus* genus by modelling the different price series and allocating the obtained wood, based on its diameter, to different industrial sectors, aiming at the fundamental adaptation to maximize the biological asset.

According to Figueiredo Filho *et al*. (2015) and Kohler *et al*. (2015), as they are classified based on the minimum diameter of use and the length of the logs, the evaluation of the forest assortment depends on the industrial purpose of the *Pinus* wood. We observe that the different price series, coming from different market segments, also have different dynamics.

The economic forecast of investment projects in planted forests requires an understanding of the market dynamics in which it operates. Simultaneously, the planning and adequacy of productive capacity must be conditioned to the silvicultural management system, as well as to the moment of implantation and commercialization of the planted forest.

In this type of segment, it is useful to predict future values based on previous data, since the prediction implies having an evolution model, even when there is randomness. According to Herrmann and Otesteanu (2016), GBM is considered a stochastic process based on random processes. However, Varnosfaderani and Verly (2017) reinforce that, to obtain more accurate results applying the GBM, the price logarithms must have a normal distribution and be independent.

Assuming independence in prices over time may impact erroneous projections of economic models, which are related to the volatility of wood products markets. FBM is also a stochastic process, and, like regular Brownian motion, it is generated by adding a series of random steps with a Gaussian distribution. However, according to Addison *et al*. (1998), in the FBM the sum is weighted over the previous Brownian steps.

As noted, in addition to the econometric analysis having evidenced the dependency structure in the data, the performance metrics of the models indicated the superiority of FBM in modelling the *Pinus* wood price series in the different assortments. In line with Zhang *et al*. (2015), as a premise, it is essential to identify the pattern of evolution of biological asset prices. We add, however, that the proper silvicultural management of forest species includes prior planning of the time horizon to be explored, through projections that, when properly measured, imply the creation of value associated with biological assets.

The consistent and exclusive potential of the Fractional Brownian Motion in modelling the prices of *Pinus* wood from different forest assortments, based on a series of two years of observations, provides important details for reproducing the results. Our experimental procedures can be revised or incorporated in other analyses considering historical data series for longer periods, mainly due to the lack of definition of the ideal size of these series.

Conclusions

Fractional Brownian Motion is the most suitable stochastic process for modelling *Pinus* wood prices for different forest assortments due to the time series showing non-stationarity and true long memory.

For *Pinus* plantation forests, the eligibility of Fractional Brownian Motion offers better accuracy for the pricing of the underlying asset, and consequently, consistent results for the valuation of real options intrinsic to forest investment projects.

The econometric analysis and stochastic modelling of forest assets from forest assortments intended for multi-products must occur for each class of tree diameter, since the economic and financial interferences are different for this sector.

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Access to data
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